



MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- *This Question Paper consists of three sections A, B and C.*
- *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
- *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
- *Section B: Internal choice has been provided in one question of two marks and one question of four marks.*
- *Section C: Internal choice has been provided in one question of two marks and one question of four marks.*
- *All working, including rough work, should be done on the same sheet as, and adjacent to*
- *the rest of the answer.*
- *The intended marks for questions or parts of questions are given in brackets [].*
- *Mathematical tables and graph papers are provided.*

SECTION A - 65 MARKS

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^{10} is:

(a) $\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$

- (b) $\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$
 (c) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
 (d) $\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix}$

(ii) Which of the following differential equations is homogeneous?

- (a) $(x^2 + y^2)dx - xydy = 0$
 (b) $(x + y)dy + (x - y)dx = 0$
 (c) $(2x + 3y)dx + (x - 2y)dy = 0$
 (d) All of the above

(iii) The function $f(x) = x^3 - 3x^2 + 4$ is:

- (a) Strictly increasing in $(0,2)$
 (b) Strictly decreasing in $(0,2)$
 (c) Neither increasing nor decreasing in $(0,2)$
 (d) Constant in $(0,2)$

(iv) $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ equals:

- (a) $\frac{\pi}{4}$
 (b) $\frac{\pi}{2}$
 (c) $\frac{\pi}{8}$
 (d) 1

(v) **Assertion:** If A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$

Reason: For independent events, $P(A|B) = P(A)$

- (a) Both Assertion and Reason are true and Reason is the correct explanation
 (b) Both Assertion and Reason are true but Reason is not the correct explanation
 (c) Assertion is true and Reason is false
 (d) Assertion is false and Reason is true

(vi) The system of equations $2x + 3y = 5$, $4x + 6y = \lambda$ has infinitely many solutions when:

- (a) $\lambda = 10$
 (b) $\lambda = 5$
 (c) $\lambda = 0$
 (d) $\lambda = -10$

(vii) A spherical balloon is being inflated at the rate of $10 \text{ cm}^3/\text{sec}$. The rate of increase of its surface area when the radius is 5 cm is:

- (a) $2 \text{ cm}^2/\text{sec}$

- (b) $4 \text{ cm}^2/\text{sec}$
 (c) $6 \text{ cm}^2/\text{sec}$
 (d) $8 \text{ cm}^2/\text{sec}$

(viii) The function $f(x) = \begin{cases} x^2 & x \leq 1 \\ 2x - 1 & x > 1 \end{cases}$ is:

- (a) Continuous but not differentiable at $x = 1$
 (b) Differentiable but not continuous at $x = 1$
 (c) Both continuous and differentiable at $x = 1$
 (d) Neither continuous nor differentiable at $x = 1$

(ix) **Assertion:** If A has 3 elements and B has 2 elements, then the number of onto functions from A to B is 6.

Reason: Number of onto functions from A to B is $2^3 - 2$ when $n(A) = 3$, $n(B) = 2$

- (a) Both Assertion and Reason are true and Reason is the correct explanation
 (b) Both Assertion and Reason are true but Reason is not the correct explanation
 (c) Assertion is true and Reason is false
 (d) Assertion is false and Reason is true

(x) A random variable X has the following probability distribution:

X	0	1	2
P(X)	0.3	0.5	0.2

Then $E(X^2)$ is:

- (a) 0.9
 (b) 1.3
 (c) 1.7
 (d) 2.1

(xi) **Statement 1:** If A is a square matrix, then $(A^T)^{-1} = (A^{-1})^T$

Statement 2: For any square matrix A, $|A^T| = |A|$

- (a) Statement 1 is true and Statement 2 is false
 (b) Statement 2 is true and Statement 1 is false
 (c) Both statements are true
 (d) Both statements are false

(xii) Find the number of equivalence relations on the set $A = \{1, 2, 3\}$

(xiii) For what value of k is the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 4 \\ 3 & 4 & 5 \end{bmatrix}$ symmetric?

(xiv) The odds against an event are 3:2. Find the probability in favour of the event.

(xv) Evaluate: $\int \frac{3x}{\sqrt{x^2+4}} dx$

Question 2

Find the point on the curve $y = x^3 - 3x$ where the tangent is parallel to the line joining (1,-2) and (2,2). [2]

Question 3

Solve: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$ [2]

Question 4

(i) If $y = (\sin x)^x$, find $\frac{dy}{dx}$

OR

(ii) Find the intervals in which $f(x) = 2x^3 - 9x^2 + 12x + 5$ is strictly increasing. [2]

Question 5

Three friends purchase fruits from a market. Their purchases (in kg) and prices (per kg) are:

Friend	Apples	Oranges	Bananas
A	2	3	1
B	1	2	3
C	3	1	2

Prices: Apple = ₹50, Orange = ₹30, Banana = ₹20

Using matrices, find the total expenditure of each friend. [2]

Question 6

(i) Differentiate: $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$ with respect to x

OR

(ii) Show that $y = e^x (A \cos x + B \sin x)$ is a solution of $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$ [2]

Question 7

Using properties of determinants, prove that:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

Question 8 [4]: If $y = e^{2x}(A \cos 3x + B \sin 3x)$, prove that $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0$.

Question 9 [4]:

(i) Solve $\frac{dy}{dx} = \frac{y}{x} + \sin \left(\frac{y}{x} \right)$.

OR

(ii) Solve $(x^2 + 1) \frac{dy}{dx} + 2xy = 4x^2$.

Question 10

Two teams (A and B) toss coins. In each round, all 4 players (2 per team) toss. If team A has both heads and team B doesn't, A wins instantly. Probability per round for A to win is $p = 9/64$. They play rounds until A wins.

(a) Probability A wins in one round?

(b) Probability the game lasts exactly 3 rounds?

OR

A disease has three risk groups in a population:

- High risk: 10% of population, disease prevalence 20%
- Medium risk: 30% of population, disease prevalence 5%
- Low risk: 60% of population, disease prevalence 1%

A person tests positive for the disease.

(a) Represent with events.

(b) Using Bayes' Theorem, find the probability the person is from the High-risk

group given they have the disease.

(c) Which risk group is most likely given disease status?

Question 11

Three vendors X, Y, Z sold notebooks, pens, and markers for a school charity. Vendor X sold 10 notebooks, 5 pens, and 15 markers for a total of ₹700. Vendor Y sold 4 notebooks, 10 pens, and 10 markers for ₹480. Vendor Z sold 6 notebooks, 5 pens, and 12 markers for ₹530.

(i) Formulate a system of linear equations for the prices of one notebook, one pen, and one marker.

(ii) Solve this system using a matrix method.

(iii) Find the price of one notebook, one pen, and one marker. [6]

Question 12

$$1. \int \frac{x^2+3x+5}{(x+1)(x^2+4)} dx$$

OR

$$1. \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\tan x} dx$$

Question 13 [6]

A closed right circular cylindrical water tank has volume 314π cubic units. If the radius is r and height is h , express h in terms of r . Then express the total surface area S (including top and bottom) in terms of r . Determine the value of r that minimizes S , and find the corresponding height. [6]

OR

(ii) A projectile is launched vertically upwards. Its height (in meters) at time t seconds is given by $h(t) = 40t - 4.9t^2$.

(a) Find the time at which the projectile reaches its maximum height.

(b) Determine the maximum height attained.

(c) Find the time when the projectile returns to the ground (height 0). [6]

Question 14 [6]:

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SECTION B - 15 MARKS

Question 15 [5]:

(i) Statement 1: The cross product of two parallel vectors is zero.

Statement 2: The dot product of two perpendicular vectors is zero.

- (a) S1 true, S2 false
- (b) S2 true, S1 false
- (c) Both true
- (d) Both false

(ii) The direction ratios of a line perpendicular to the plane $2x - 3y + 4z = 5$ are:

- (a) $\langle 2, -3, 4 \rangle$
- (b) $\langle 1, 1, 1 \rangle$
- (c) $\langle -2, 3, -4 \rangle$
- (d) $\langle 0, 0, 1 \rangle$

(iii) If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then:

- (a) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$
- (b) $\vec{a} \perp \vec{b}$
- (c) $\vec{a} \parallel \vec{b}$
- (d) None

(iv) The angle between $\vec{i} + \vec{j} + \vec{k}$ and $\vec{i}$ is:

- (a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{3}$

(v) Find $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $\vec{a} \cdot \vec{b} = 5$.

Question 16 [2]:

(i) Let $A(1,2,3)$, $B(4,5,6)$, $C(7,8,9)$. Find the area of triangle ABC.

OR

(ii) Find x if vectors $\vec{p} = x\vec{i} + \vec{j} + \vec{k}$ and $\vec{q} = \vec{i} + x\vec{j} + \vec{k}$ are perpendicular.

Question 17 [4]:

(i) Find the equation of the plane through (1,2,3) and perpendicular to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

(OR)

(ii) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$.

Question 18 [4]: Find the area bounded by $y = x^2$, x -axis, and lines $x = 1$, $x = 3$.

SECTION C – 15 Marks

(Attempt all questions in this section.)

Question 19 [5]: In subparts (i) and (ii) choose the correct option, and in subparts (iii) to (v) answer as instructed. [5 × 1 = 5]

(i) If the **Average Cost (AC)** of production is constant at all output levels, which is true?

(a) $\text{Marginal Cost} > AC$ (b) $MC = AC$ (c) $MC < AC$ (d) $MC = (1/2)AC$

(ii) Statements about regression coefficients:

Statement 1: It measures the degree of linear relationship between two variables.

Statement 2: It indicates change in one variable for unit change in the other.

Which is correct? (Recall: regression vs correlation)

(a) S1 true, S2 false (b) S2 true, S1 false

(c) Both true (d) Both false

(iii) The means are $\bar{x} = 53$, $\bar{y} = 28$. The regression coefficients are $b_{y|x} = 0.6$ and $b_{x|y} = 0.4$. Find the correlation coefficient r . [1]

(iv) The total revenue from selling x units is $R(x) = 4x^2 + 20x + 100$. Find the marginal revenue at $x = 10$. [1]

(v) A company's cost function is $C(x) = 150x + 6000$. If each item sells for ₹250, find the minimum number of items that must be sold daily to break even. [1]

Question 20 [2]:

(i) A service company's monthly cost for x units is given by Fixed cost = ₹4000 and Variable cost = ₹ $(20x - x^2)$. What value of x minimizes the total cost? [2]

(Or)

(ii) A monopolist's demand function is $x = 80 - 2p$. Determine the quantity x at which the marginal revenue is zero. [2]

Question 21 [4]: (Application)

A survey of 50 families studied expenditure on accommodation ($\text{₹}x$) and on food/entertainment ($\text{₹}y$), yielding: $\sum x = 5000$, $\sum y = 7500$, $\sigma_x = 50$, $\sigma_y = 30$, and $r = 0.6$. Using this data, estimate the expenditure y when $x = 120$. [4]

Question 22 [4]: (Linear Programming)

(i) Maximize $Z = 5x + 4y$ subject to

$$x + y \leq 40, \quad 2x + y \leq 60, \quad x \geq 0, \quad y \geq 0.$$

(a) Find the corner points of the feasible region.

(b) Determine the maximum value of Z and the point (x, y) at which it occurs. [4]

OR

(ii) A feasible region is bounded by $x \geq 0$, $y \geq 0$, $x + 2y \leq 12$, $3x + y \leq 18$.

(a) Find the corner points of this region.

(b) Maximize $Z = 3x + 5y$. [4]

Q14

In a school, three subject teachers English, Math, and Science sometimes give surprise tests on the same day. Based on past records:

- The English teacher gives a test 90% of the time
- The Math teacher gives a test 80% of the time
- The Science teacher gives a test 70% of the time

Each teacher decides independently. If the average number of surprise tests is less than 2.3 then the teachers should coordinate better to increase the performance of the students. Otherwise, no action is needed.

Let X be the number of surprise tests a student gets on a given day. So, $X \in \{0, 1, 2, 3\}$.

- Find the probability for each possible number of surprise tests.
- Use the probabilities to build a distribution table.
- Calculate the average number of surprise tests per day.
- Based on your calculations, decide: Should the teachers coordinate better? Or is the current plan acceptable?